Logic, Reasoning, and Persuasion, Week 11-1 Handout

1 | M&Ms

Exercise 4: m&ms in a Bag

I have a collection of half blue m&ms and half yellow m&ms. I take an m&m out randomly and place it in a bag.

Q1 What is the probability that the m&m in the bag is yellow?

(Carroll 1958) I take another yellow m&m from the collection, and place it in the bag. I shake the bag, and randomly take one of the two m&ms out.

Q1.1 What is the probability that the m&m I take out is yellow?

Suppose that I look at the color and it turns out to be yellow.

Q2 Should you *increase* or *decrease* your confidence that the remaining m&m in the bag is yellow, or keep it the same?

Q3 What is the probability that the remaining m&m in the bag is yellow?

To help with **Q2** and **Q3**, let's ask some further questions. Let

$$E = \text{the m\&m I randomly take out is yellow.}$$

Call the first m&m I put in m&m1, and the second one m&m2. Now let

$$YY = \text{m\&m1 is yellow and m\&m2 is yellow}$$
$$BY = \text{m\&m1 is blue and m\&m2 is yellow.}$$

Based on the setup of the case, we know that exactly one of YY and BY is true.

1. **For Q2**: If I randomly take one of the two m&ms out, and learn that E (that the m&m I took out is yellow), is this evidence for or against YY? (a) and (b) below help you figure this out exactly, but even without doing the calculations, do you think that $\Pr(E \mid YY) > \Pr(E)$ or not?
   (a) First figure out $\Pr(E)$ using the law of total probability:
   $$\Pr(E) = \Pr(E \mid YY)\Pr(YY) + \Pr(E \mid BY)\Pr(BY) =$$
   (b) Then use the *Evidence-For* lemma, checking if $\Pr(E \mid YY) > \Pr(E)$.
2. **For Q3**: Use Bayes' Theorem to find $\Pr(YY \mid E)$.

Exercise 5: m&ms in a Bag 2

After taking out the m&m (which turned out to be yellow) from the bag , I get another bag and put a yellow m&m in along with two blue m&ms. Now, what gives you a better chance of drawing a yellow m&m? (Also Carroll (1958)).

1. Randomly choose one of the bags, and then draw a m&m from that bag.
2. Pour the m&ms into the same bag, and draw a m&m from that bag.

To figure out what gives us the better chances, let's figure out the probability of drawing a yellow m&m in each case.

Option 1: Randomly Choose a Bag, and then Draw from that Bag.
If you do this, then $\Pr(\text{Bag }1) =$ ____, and $\Pr(\text{Bag }2) =$ ____.

By total probability, if you've taken Option 1, then

$$\begin{aligned}\Pr(\text{draw Y}) &= \Pr(\text{draw Y} \mid \text{Bag 1})\Pr(\text{Bag 1}) + \Pr(\text{draw Y} \mid \text{Bag 2})\Pr(\text{Bag 2})\\ &=\end{aligned}$$

Option 2: Pour the m&ms into the same bag
If you do this, then the resulting bag has the yellow m&m and two blue m&ms from Bag 2, and the m&m from Bag 1, which is either blue or yellow. Thus $\Pr(3\text{B}, 1\text{Y}) =$ ____ and $\Pr(2\text{B}, 2\text{Y}) =$ ____.

So by total probability, if you've taken Option 2, then

$$\begin{aligned}\Pr(\text{draw Y}) &= \Pr(\text{draw Y} \mid 3\text{B}, 1\text{Y})\Pr(3\text{B}, 1\text{Y}) + \Pr(\text{draw Y} \mid 2\text{B}, 2\text{Y})\Pr(2\text{B}, 2\text{Y})\\ &=\end{aligned}$$

2 | Independence

Recall the following:

1. **Definition:** E is *evidence for* H if $\Pr(H \mid E) > \Pr(H)$.
2. **Lemma:** $\Pr(H \mid E) > \Pr(H) \iff \Pr(E \mid H) > \Pr(E)$.

If E is neither evidence *for* nor against H, then we say H is *independent of* E.

Definition of Independence

H is **independent of** E if $\Pr(H \mid E) = \Pr(H)$.

Sometimes people say two events A and B are independent. Intuitively, this is true if A and B have *nothing to do with each other*, so that learning that A happened is irrelevant to your guess of whether B happened, and vice-versa.

Exercise 1: For each of the following cases, is E evidence for or against H, or is H independent of E?

1. $\Pr(E \mid H) = 0.5$ and $\Pr(E) = 0.4$.
2. $\Pr(E \mid H) = 0.5$, $\Pr(H) = 0.8$, and $Pr(E) = 0.9$.
3. $\Pr(E \mid H) = 0.5$, $\Pr(H \mid E) = 0.7$, and $Pr(H) = 0.6$.
4. $\Pr(E \mid H) = 0.5$, $\Pr(H \mid E) = 0.7$, and $Pr(H) = 0.7$.

Exercise 2: I flip a fair coin three times. Let A = the first coin comes up heads and B = the second coin comes up heads. Are A and B independent?
Exercise 3: Let D = the first two coins come up heads. Are A and D independent?