The Evidence Function

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This handout with references:
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The function of evidence is to be a guide to the truth. I argue that in standard *epistemic setups* this implies FUNCTION ACCESS: rational epistemic agents have access to the function that specifies *what* evidence they have in *what* conditions. This implies EVIDENCE INTERNALISM: rational epistemic agents are certain what their evidence is.

0. The Evidence

COIN: At time t_1, K walks into a room, and either sees a hedgehog or a tortoise. At time t_0, a fair coin had been flipped, and the hedgehog placed if it came up heads, the tortoise placed if it came up tails. Based on K's prior opinions at t_0 and what they see at t_1, K updates their opinions on whether the coin came up heads or tails at t_0.

The Evidence Setup: a triple $\langle C, E, f \rangle$:

1. *Worldly possibilities* $C = \{c_1, c_2, \ldots\}$ (finite). Interpretation: the possibilities that might actually be the case, at appropriate level of grain. The possibilities are not time-indexed.
2. *Evidence possibilities* $E = \{e_1, e_2, \ldots\}$ (finite). Interpretation: "things that might be K's evidence." E is time-indexed to t_1.
3. *Evidence function* $f : C \to E$, which describes what evidence an agent gets in different situations. Interpretation: for every possible condition $c \in C$, the evidence function returns the evidence $f(c)$ that the agent gets if c is actual.

The Evidence Setup in COIN:

1. $C = \{c_H, c_T\}$. c_H is the possibility in which at t_0 the coin comes up heads and a hedgehog placed in the room, and at t_1 K sees the hedgehog. c_T is the possibility in which at t_0 the coin comes up tails and a tortoise placed in the room, and at t_1 K sees the tortoise.
2. $E = \{e_H, e_T\}$. Either K sees a hedgehog (K's evidence is e_H) or K sees a tortoise (K's evidence is e_T).
3. K sees a hedgehog at t_1 (evidence e_H) if the coin comes up heads and a hedgehog is placed at t_0 (c_H). They see a tortoise at t_1 (evidence e_T) if the coin comes up tails and a tortoise is placed at t_0 (c_T). Thus $f(c_H) = e_H$ and $f(c_T) = e_T$.

Priors and Updates: Given an evidence setup $\langle C, E, f \rangle$, an epistemic agent can be characterized with a prior and an update rule:

4. A *prior credence function* π, which maps each set of conditions $C_i \subseteq C$ to a real number $x \in [0, 1]$, to be interpreted as K's degree of confidence that one of the conditions $c \in C_i$ is actual. I'll assume π is probabilistic.[1]
5. An *update rule* u, which takes as argument a credence function π and some $e \in E$ and returns another credence function $u[\pi, e]$.

$\to$ In all: let an **Epistemic Setup** $S = \langle \langle C, E, f \rangle, \pi, u \rangle$ be a tuple of an evidence setup, a prior, and an update.[2]

[1] Let Cr be the set of all probability functions on the powerset $\mathcal{P}(C) =_{\text{df}} \{C_i \subseteq C\}$ of C.

[2] Equivalent frameworks are ubiquitous in the updating literature: e.g. in Greaves & Wallace they are called "experiments", in Schoenfield and Zendejas Medina "learning experiences", in Schultheis "learning situations", in Gallow "learning scenarios".

This Talk: – I defend FUNCTION ACCESS: *An ideally rational epistemic agent in an epistemic setup has access to their evidence function.*

- **Why is Function Access true?** I argue: in epistemic setups, FUNCTION ACCESS is unavoidable, *given our concept of evidence*.
- **What's at stake?** In epistemic setups, FUNCTION ACCESS implies EVIDENCE INTERNALISM: if an ideal epistemic agent K in a setup $\langle C, E, f \rangle$ has evidence that warrants opinion o, then their evidence warrants certainty that their evidence warrants opinion o.[3]
- **So is internalism true?** Maybe. But another conclusion is that the framework of epistemic setups does not have the resources to capture externalist intuitions. We may need a rethinking of evidence, *outside* of the standard modeling of epistemic setups.

[3] More carefully: for all epistemic setups S, if u is ideally rational, then for all $e \in E$, $u[\pi, e]$ ($[\![$my evidence is $\mathbf{e}]\!]) = 1$.

1. *The Argument for Function Access $\to$ Evidence Internalism*

The **condition function** gives the set of conditions that the evidence function maps to a particular evidence proposition:

$$f^\star(e) =_{\text{df}} \{c \in C \mid f(c) = e\}. \qquad \text{(The Condition Function)}$$

So $f^\star(e)$ is a reasonable formal interpretation of $[\![$my evidence is $\mathbf{e}]\!]$.

Say that an agent K in an setup $\langle\langle C, E, f\rangle, \pi, u\rangle$ **has access to the evidence function** f of their setup if the update u can include f or $f^\star$ either (1) in the definition of u or (2) in an argument passed to u.

Much further argumentation will depend on this update rule:[4]

$$u[cr, e](\,\cdot\,) = cr\,(\,\cdot\,\mid f^\star(e)) \qquad (\star\textsc{cond})$$

[4] where $cr\,(\,\cdot\,\mid f^\star(e)) =_{\text{df}} \frac{cr(\,\cdot\,\wedge\bigcup f^\star(e))}{cr(\bigcup f^\star(e))}$
$\star$COND is from Schoenfield (2017).

The Argument for FUNCTION ACCESS $\to$ EVIDENCE INTERNALISM

(P1.1) If K has access to f, then $\star\textsc{cond}[\pi, e]\,([\![$my evidence is $\mathbf{e}]\!]) = 1$.

(P1.2) If K has access to f, then $\star$COND is the rational update rule.[5]

(C1) Therefore, if K has access to f, then if u is rational, then $u[\pi, e]\,([\![$my evidence is $\mathbf{e}]\!]) = 1$.

[5] the *unique* rational update rule, given uniqueness (vs permissivism)

Defense of (P1.1): If f is accessible, then $\star$COND is an admissible update function and $f^\star$ is allowed to be passed as argument to u. In particular, we can write $u[\pi, \cdot](f^\star(e))$. Then we have, for all $e \in E$,

$$\star\textsc{cond}[\pi, e]([\![\text{my evidence is } \mathbf{e}]\!]) \qquad (1)$$
$$= \star\textsc{cond}[\pi, e](f^\star(e)) = \pi(f^\star(e) \mid f^\star(e)) = 1. \qquad (2)$$

Defense of (P1.2):

- $\star$COND is the update rule that maximizes expected accuracy among available update rules with access to f.[6]
- $\star$COND is the rational update rule for K if, upon getting evidence that the true condition is in C_i, K cares about accuracy only in C_i.[7]
- $\star$COND is more accurate *in every possibility* than any other update rule with access to f.[8]

[6] Implied by Greaves & Wallace (2006).

[7] Implied by Gallow (2014).

[8] Implied by Briggs & Pettigrew (2020).

2. *The Argument for Function Access*

The Argument for FUNCTION ACCESS

(P2.1) Something can be evidence only if it can count as evidence for a rational epistemic agent.

(P2.2) Something can count as evidence for a rational epistemic agent in a setup S only if it can be characterized as the output of an evidence function f that the agent can access.

(C2) Therefore, a rational epistemic agent in a setup S always has access to the evidence function f for their evidence in S.

Defense of (P2.1): The *function* of evidence, in our term of art, is to be *the thing that an epistemic agent responds to*, rationally or not, in updating their opinions to be closer to the truth.[9,10]

Defense of (P2.2): The only way for something to be intelligible to an epistemic agent as evidence is for it to be intelligible to an epistemic agent[11] *in the guise of* being the output of the evidence function – that is, indicating that some conditions $\{c_1, c_2, \ldots\} = f^\star(e)$ are true.

- In Coin: informally, the sense data that K gets is only evidence if it is evidence *for* some set of conditions being true.
- Suppose a random proposition is flung at K (metaphorically): unless this proposition has some nontrivial connection to the truths in K's situation[12], we have no good story about why K should adopt any particular response to it.
- In an epistemic setup, *being the output of the evidence function* is what makes something *evidence* at all, as opposed to some random proposition, or some random sense data, or some random perturbations of one's neural states.

[9] I recognize this is too vague.

[10] An analogous argument: something can't count as a *language* if even ideal communicators couldn't use it.

[11] It's hard not to make this too person-level: but really all I need is to that it has to be sensible to plug the evidence, whatever it is, into a proposed epistemic update procedure.

[12] And let's not beg the question by saying it's K's evidence or is the strongest thing they learn

3. The Defense of Function Access against Bayesianism

Bayesians characterize evidence in the form of a *proposition*: thus $E = \{e_1, e_2, \ldots\}$ where each $e_i \subseteq C$. So then the *propositional content* of each evidence possibility e is itself something that an agent could suppose is true. Bayesians say rational agents update by *conditionalization*:

$$u[\pi, e](\cdot) = \pi(\cdot \mid e), \qquad (e\text{COND})$$

- *Internalist* Bayesians often assume that epistemic agents learn in advance *exactly which member of some specified partition of C is true (and no more).*[13] In these circumstances, eCOND and ⋆COND are equivalent,[14] so any defense of ⋆COND is a defense of eCOND.
- *Externalist* Bayesians say that E might not partition C. But in any case where eCOND and ⋆COND come apart, ⋆COND does strictly better on accuracy grounds than eCOND. So the only way to defend eCOND against ⋆COND is to disallow ⋆COND.

[13] That is, E partitions C: every $c \in C$ is in a member of *exactly one* $e \in E$. This specification makes the propositional evidence *factive* ($\forall e \in E : f^\star(e) \subseteq e$) and *transparent* ($\forall e \in E : e \subseteq f^\star(e)$), and thus means that for all $e \in E$, $e = f^\star(e)$.

[14] Just substitute in $f^\star(e)$ for e.

Externalist Bayesian Arguments for *e*cond over ⋆cond: The externalist could say that e is the strongest thing you learn, and that the iteration principle **LL** is false: when K learns e, K does not learn ⟦I learned that **e**⟧. Then they can say we should evaluate rational agents based on what they learn, not propositions that are true when they learn what they learn.[15]

$\rightarrow$ **Problem**: how is "learning" and "the strongest thing learned" characterized? Is it a primitive or is it defined in other terms?

1. If "learning" is a primitive, then in the absence of a separate argument that the evidence function is inaccessible, the externalist cannot block the ⋆COND update.

[15] Zendejas Medina (2024)

2. If "learning" is not a primitive, then the terms in which it's defined must be defended against ⋆COND. E.g.: if "what you learn" is "the strongest proposition that warrants credence 1", then the externalist has to say why what you learn is e and not $f^\star(e)$.

Stepping Back: In the Bayesian framework, when an agent gets evidence e, there are two propositions: the proposition e and the proposition $f^\star(e)$. There is a natural explanation of why $f^\star(e)$ has evidential import: *it specifies the true conditions*. There is no natural, non-primitive explanation of why e has evidential import. When $e = f^\star(e)$, this is not a problem. But otherwise, the propositional content of e seems to be a mere confusion of additional machinery.

Where $\mathbb{L}E$ is the proposition that K learned that E (this corresponds to $f^\star(e)$, Zendejas Medina compares:

- EPISTEMIC ADMISSIBILITY: after learning that E, a rational agent will implement the antecedently best actionable plan for what to do or believe if E is true.
- AUTO–EPISTEMIC ADMISSIBILITY: after learning that E, a rational agent will implement the antecedently best actionable plan for what to do or believe if they learn that E.

He argues for AUTO-EPISTEMIC ADMISSIBILITY: you are required to implement a plan with condition c only if you learn c. When $\mathbb{L}E$ is true, you might not have learned that $\mathbb{L}E$ is true. You learn that E is true, so you should implement the plan for if E is true. **Problem**: this requires an *antecedent* argument that $f^\star$ is not accessible. Otherwise, K can *infer* that if e is the rational condition to suppose to be true, then the actual condition must be $f^\star(e)$.

4. *The Allowable Update Functions*

The standard sandbox in which the literature on rational updating plays is the *epistemic setup*. In these update rules, *some* functions are allowed, and *some* inputs are allowed. Is there a principled way to draw the line that doesn't commit us to evidence internalism within the framework?[16] If not, it may be that to characterize evidence externalism, we need to go beyond the framework.

Example: Dorst (2020, 2023) describes update functions that are characterized manually as transitions from evidence and priors to posteriors, with a possible interpretation that the evidence *tweaks your neural states* so your credal states are different. A model:
1. $C = \{c_1, c_2\}$, $E = \{e_1, e_2\}$, $f : C \to E = [f(c_1) = e_1, f(c_2) = e_2]$.
2. π given by $(\pi(c_1), \pi(c_2)) = (1/2, 1/2)$.
3. $u[\pi, \cdot] : E \to \mathbb{C}$ given by

$$u[\pi, e_1] = (2/3, 1/3); \quad u[\pi, e_2] = (1/3, 2/3).$$

Is there a principled line that allows the model above but not ⋆COND?[17]

[16] **A recursive attempt**:
(i) If $e \in E$ is one's evidence and $f : E \to X$ is an allowable function, then $f(e)$ is an allowable output.
(ii) If f_1 and f_2 are allowable functions, then the partial composition $f_1|_{x_I = f_2}$, obtained by replacing the arguments in indices I with the output of $f_2(\cdot)$, is allowed.
(iii) The prior $\pi : \mathcal{P}(C) \to [0, 1]$ is an allowable function.
(iv) The arithmetic operations $+, -, \cdot, \div$ and the set-theoretic operators $\cup, \cap, \setminus$ are allowable functions.
(v) Other allowed functions?

[17] Some issues:
- This model uses (i)+(ii) above.
- This model is meant to be higher-order uncertain: $f(c_1) = e_1$, so in condition c_1, the posterior is $u[\pi, e_1]$, and $u[\pi, e_1](c_1) = 2/3$. So if we can write $u[\pi, e_1](⟦\text{My evidence is } e_1⟧) = u[\pi, e_1](f^\star(e_1)) = u[\pi, e_1](c_1) = 2/3$, then we can characterize the uncertainty formally. But this requires $f^\star$ to be allowed!
- And given (i), (ii), and $f^\star$, all we need is (iii) and (iv) to get ⋆COND.

5. *The Import of Evidence*

- Within the tractable worlds of epistemic setups, I think it's hard to escape evidence internalism, because *being the output of the evidence function* seems to be what makes evidence *evidence*.
- But evidential nontransparency, inexact learning, and rational uncertainty seem ubiquitous. So perhaps to formally model these phenomena we need to leave the confines of epistemic setups.
- Outside of evidence setups, however, our picture of the evidential situations of epistemic agents, and thus our theory of what explains the *import* of evidence, might look quite different.

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